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UNIVERSAL PHYSICAL EQUATIONS OF TURBULENCE

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【Synopsis of the full paper in Chinese pp. 199～203】

Using the rate of deformation tensor and the perturbed deformation displacement to decompose the fluctuating velocity of turbulent flow, and then using density-weighted off-centre ensemble average method, instead of the conventional Reynolds average, the N－S equation has been averaged in a new way without introducing any physical hypothesis. The unknown correlation terms only appear at terms with third or higher order of accuracy. Therefore after cutting off the correlation terms, a set of closed equations of turbulent flow with secondary order of accuracy has been derived. There is no correlation terms in the equations, no any empirical coefficients, and no need for performing any kind of turbulence models. Any complex turbulent flow under Mach number $Ma = 3.6$ can be described with high order of accuracy by the equations and most of physical characteristics of turbulence would be explained by the equations. In fact, the set of equations are the physical equations of turbulence. Both of the equations for incompressible or compressible flow are introduced. The equations for the incompressible turbulent flow are as follows:

$$\frac{\partial \bar{\boldsymbol{V}}}{\partial t} + (\bar{\boldsymbol{V}} \cdot \nabla)\bar{\boldsymbol{V}} = -\frac{1}{\rho}\nabla\hat{P} + \nu_e \nabla^2 \hat{\boldsymbol{V}} + (\bar{\boldsymbol{v}}' \cdot \nabla)\hat{\boldsymbol{V}} + \frac{\nu_e}{2}[\nabla(\overline{\delta \boldsymbol{r}} \cdot \nabla)] \times \hat{\Omega} \tag{1}$$

$$\frac{\partial \bar{\boldsymbol{v}}'}{\partial t} + (\bar{\boldsymbol{V}} \cdot \nabla)\bar{\boldsymbol{v}}' = -\frac{1}{\rho}\nabla\bar{p}' + \nu_e \nabla^2 \bar{\boldsymbol{v}}' - (\bar{\boldsymbol{v}}' \cdot \nabla)\bar{\boldsymbol{V}} + \nabla K \tag{2}$$

$$\nabla \cdot \bar{\boldsymbol{V}} = 0 \tag{3}$$

$$\nabla \cdot \bar{\boldsymbol{v}}' = 0 \tag{4}$$

Where $\boldsymbol{V} = \bar{\boldsymbol{V}} - \bar{\boldsymbol{v}}'$, $\hat{P} = \bar{P} - \bar{p}'$, $\hat{\boldsymbol{V}} = \boldsymbol{V} - \overline{\nabla\Phi}$, $\hat{\Omega} = -\nabla \times \boldsymbol{V}$, $K \cong \frac{1}{2}\bar{\boldsymbol{v}}' \cdot \bar{\boldsymbol{v}}'$

$$\nu_e = \nu_L + \nu_T, \quad \nu_T = -\nu_L \frac{\nabla^2(\overline{\nabla\Phi})}{\nabla^2 \boldsymbol{V}}, \quad \bar{\boldsymbol{v}}' = \nabla\boldsymbol{V} \cdot \overline{\delta\boldsymbol{r}} = \overline{\nabla\Phi} + \frac{1}{2}\hat{\Omega} \times \overline{\delta\boldsymbol{r}}$$

where $\bar{\boldsymbol{V}}$, $\bar{P}$ are mean flow parameters, $\bar{\boldsymbol{v}}'$, $\bar{p}'$ are averaged fluctuating parameters ($\bar{\boldsymbol{v}}' \neq 0$ for the special average), and $\boldsymbol{V}$, $\hat{P}$ are mean flow minus mean pulse parameters. $\overline{\delta\boldsymbol{r}}$ is mean perturbed deformation displacement, $\overline{\nabla\Phi}$ is mean perturbed strain velocity, and $(1/2)\ \hat{\Omega} \times \overline{\delta\boldsymbol{r}}$ is mean perturbed rotation velocity.

The equations for the compressible turbulent flow are as follows:

$$\frac{\partial \bar{\boldsymbol{V}}^o}{\partial t} + (\bar{\boldsymbol{V}}^o \cdot \nabla)\bar{\boldsymbol{V}}^o = -\frac{1}{\rho}\nabla\hat{P} + \nu_e \nabla^2 \hat{\boldsymbol{V}} + (\bar{\boldsymbol{v}}^{o\prime} \cdot \nabla)\hat{\boldsymbol{V}} + \frac{\nu_e}{2}[\nabla(\overline{\delta\boldsymbol{r}} \cdot \nabla)] \times \hat{\Omega}$$
$$+ \frac{\nu_e}{3}\nabla(\nabla \cdot \hat{\boldsymbol{V}}) \tag{5}$$

or
$$\frac{\partial \bar{u}_i^o}{\partial t} + \bar{u}_j^o \frac{\partial \bar{u}_i^o}{\partial x_j} = -\frac{1}{\rho}\frac{\partial}{\partial x_i}\hat{P} + \frac{\partial}{\partial x_j}\nu_e\left(\frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i}\right) + \bar{u}_j^{o\prime}\frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial}{\partial x_j}\frac{\nu_e}{2}\overline{\delta r}_m E_{ijk}\frac{\partial \hat{\Omega}_k}{\partial x_m} - \frac{2}{3}\frac{\partial}{\partial x_j}\nu_e\delta_{ij}\frac{\partial \hat{u}_k}{\partial x_k}$$

$$\frac{\partial \bar{\boldsymbol{v}}^{o\prime}}{\partial t} + (\bar{\boldsymbol{V}} \cdot \nabla)\bar{\boldsymbol{v}}^{o\prime} = -\frac{1}{\rho}\nabla\bar{p}' + \nu_e\nabla^2\bar{\boldsymbol{v}}' + \frac{\nu_e}{3}\nabla(\nabla \cdot \bar{\boldsymbol{v}}') - (\bar{\boldsymbol{v}}^{o\prime} \cdot \nabla)\bar{\boldsymbol{V}} + \nabla K^o \tag{6}$$

or
$$\frac{\partial u_i^{o\prime}}{\partial t} + \bar{u}_j \frac{\partial u_i^{o\prime}}{\partial x_j} = -\frac{1}{\bar\rho}\frac{\partial \bar p'}{\partial x_i} + \frac{\partial}{\partial x_j}\nu_e\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right) - \bar u_j^{o\prime}\frac{\partial u_i}{\partial x_j} + \frac{\partial K^o}{\partial x_i} - \frac{2}{3}\frac{\partial}{\partial x_j}\nu_e\delta_{ij}\frac{\partial u_k'}{\partial x_k}$$

$$\frac{\partial \rho}{\partial t} + \bar u_j \frac{\partial \rho}{\partial x_j} + \bar\rho\frac{\partial u_k}{\partial x_k} = 0 \tag{7}$$

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar\rho \bar u_j' + \bar\rho' \bar u_j - \bar\rho' \bar u_j^{o\prime}\right) = 0 \tag{8}$$

$$\frac{\partial h^o}{\partial t} + \bar u_j^o \frac{\partial h^o}{\partial x_j} = Q + \frac{1}{\bar\rho}\left(\frac{\partial \hat P}{\partial t} + \bar u_j\frac{\partial \hat P}{\partial x_j}\right) + \frac{1}{\bar\rho}\frac{\partial}{\partial x_j}\lambda\frac{\partial T}{\partial x_j} + \frac{\hat\Phi}{\bar\rho} + \bar u_j^{o\prime}\frac{\partial}{\partial x_j}\left(\bar h - \bar h^{o\prime}\right) \tag{9}$$

$$\hat\Phi = \frac{1}{2}\mu_e\left[\left(\frac{\partial \hat u_i}{\partial x_j} + \frac{\partial \hat u_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}\frac{\partial \hat u_k}{\partial x_k}\right]\left(\frac{\partial \hat u_i}{\partial x_j} + \frac{\partial \hat u_j}{\partial x_i}\right)$$

$$\frac{\partial h^{o\prime}}{\partial t} + \bar u_j \frac{\partial h^{o\prime}}{\partial x_j} = \frac{1}{\bar\rho}\left(\frac{\partial \bar p'}{\partial t} + u_j\frac{\partial \bar p'}{\partial x_j}\right) + \frac{1}{\bar\rho}\frac{\partial}{\partial x_j}\lambda\frac{\partial T'}{\partial x_j} + \frac{\bar\Phi'}{\bar\rho} - \bar u_j^{o\prime}\frac{\partial}{\partial x_j}\left(\bar h - \bar h^{o\prime}\right) \tag{10}$$

$$\bar\Phi' = \frac{1}{2}\mu_e\left[\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}\frac{\partial u_k'}{\partial x_k}\right]\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\bar P = \bar\rho R \bar T^o \tag{11}$$

$$\bar p' = R\left(\bar\rho' \bar T + \bar\rho \bar T' - \bar\rho' \bar T^{o\prime}\right) \tag{12}$$

where $\bar{\boldsymbol V}, \bar P, \bar T, \bar\rho, \bar h$ are non-density-weighted mean flow parameters, $\bar{\boldsymbol v}', \bar p', \bar T', \bar\rho'$ are averaged non-density-weighted fluctuating parameters, and $\boldsymbol V$, P, T, ρ, h are non-density-weighted mean flow minus mean pulse parameters.

$$\boldsymbol V = \bar{\boldsymbol V} - \bar{\boldsymbol v}', \quad P = \bar P - \bar p', \quad T = \bar T - \bar T', \quad \rho = \bar\rho - \bar\rho'$$

$$\bar{\boldsymbol v}' = \overline{\nabla \boldsymbol V \cdot \delta \boldsymbol r} = \overline{\nabla\Phi} + \frac{1}{2}\hat\Omega \times \overline{\delta \boldsymbol r}, \quad \hat\Omega = -\nabla \times \boldsymbol V, \quad \hat{\boldsymbol V} = \bar{\boldsymbol V} - \overline{\nabla\Phi}$$

$$\hat P = \bar P - \bar p', \quad \nu_e = \nu_L + \nu_T, \quad \nu_T = -\nu_L\frac{\nabla^2(\overline{\nabla\Phi})}{\nabla^2\bar{\boldsymbol V}}, \quad K = \frac{1}{2}\overline{\boldsymbol v' \cdot \boldsymbol v'} \cong \frac{1}{2}\bar{\boldsymbol v}' \cdot \bar{\boldsymbol v}'$$

All parameters with superscript o denote density-weighted mean parameters. The relationships between density-weighted and non-density-weighted parameters, with secondary order of accuracy, are

$$\bar{\boldsymbol V}^o = \boldsymbol V\left(1 - \frac{\bar\rho'}{\bar\rho}\right), \quad \bar v^{o\prime} = \bar v'\left(1 - \frac{\bar\rho'}{\bar\rho}\right), \quad \bar T^o = T\left(1 - \frac{\bar\rho'}{\bar\rho}\right), \quad \bar T_o' = \bar T\left(1 - \frac{\bar\rho'}{\bar\rho}\right)$$

Using long time scale, the above equations describe mean turbulent flow. Using shorter time scale, the structures of turbulent flow with corresponding scales can be described. The statistics movement and structure movement have been united by the universal physical equation of turbulence.

If let $\boldsymbol V = \bar{\boldsymbol V} - \boldsymbol v' + \boldsymbol v'' - \boldsymbol v''' + \cdots$, higher oder of accuracy for supersonic flow is possible, and then the second term on the right side of eq. (2) becomes

$$(\boldsymbol V \cdot \nabla)\left(\bar{\boldsymbol v}^{o\prime} + \frac{\bar{\boldsymbol v}^{o\prime\prime}(2\bar{\boldsymbol v}^{o\prime} - \bar{\boldsymbol v}^{o\prime\prime})}{\boldsymbol V} + \cdots\right)$$

where $\bar{\boldsymbol v}^{o\prime\prime}$ may be obtained from a similar equation of high order to eq. (6)

ENGINE FLIGHT LOAD RELIABILITY ANALYSIS

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ABSTRACT "Engine Flight Load Reliability Analysis" is a special reliablity research on the subject of engine integral load chart based on "JX/TJ (Trubojet) X Engine Flight Load Mesurement". This paper makes full scale statistics and analysis on a great number of data obtained form the actual measurements of flight testing and quite a number of data collected from flight-line service. A theortical research has been made on application of double design works to engine low cycle fatigue life calculation and testing. The results of this research have been proved by the statistics and analysis of JX/TJX Engine Flying Testing Measurements.